CHAPTER VI

FACTOR ANALYSIS AND INTERPRETATION OF FACTORS

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Introduction

Factor Analysis, like all other statistics, is a branch of applied mathematics which is used largely as a tool to provide a mathematical model to explain the underlying behaviour of the data (Herman, 1960). "Factor Analysis is a statistical technique for reducing or simplifying a large number of variates in experimental data to a smaller number of hypothetical variates that represent weighted sum of the observed variates" (The New Encyclopaedia Britannica, 1979). It involves the analysis of a large battery of tests in order to identify a few common factors. Thus, the tests which best measure these factors may be considered direct measure of the factor's mind. The principle concern of factor analysis, therefore, is the resolution of a set of variables linearly, the individual factor loadings of the various tests included

in a battery on the factors provide mathematical information about the behavioural composition of the tests and are, thus, the source of direct and concrete evidence of the tests' validity. The factor analysis is, thus, a highly mathematical technique as well as advanced educational technology.

Regarding the operational aspect of factor analysis, the British School of Thought and the American School of Thought are the two schools of thought and believe in the Hierarchical Group Factory Theory and the Multiple Factor Theory respectively. Holzinger and Herman (1938) and Eysenck (1939) demonstrated quite early that the contents of group factors correspond very closely to that of multiple factors. As this study drew inspiration from the British school of thought, the technique chosen for factor analysis was the principal Axes Method which has been employed in the present study.

Some Background Studies

Vaidya (1964) investigated problem solving among certain group of adolescent pupils using questionnaire approach as well as individual approach, with the help of Piagetian type tasks and other variables : intelligence, age 11^{+} test scores on Arithmatic and English, persistence, maladjustment and interest in things in Central London. The data when factorially analysed by Hotelling method, revealed ten factors of which only the four factors namely, Attainment factor, Practical factor, Interest factor and Adjustment factor, were

found significant and were interpreted after rotation. Vaidya (1975) some years later investigating a more elaborate study, using 45 variables on the growth of logical thinking during adolescence using seventeen problems and other tests, like intelligence, adjustment etc., on a sample of 200 students of grade VI to X found ten factors : Schematic learning, General adjustment, Problem orientation, Sensing problems, Symbolisation, Testing hypotheses, Using constant differences, Aspect character, Seeing the problem as a whole and Intelligence.

Sandhu (1980) investigated the structure of formal thought with the help of ten Piagetian type tasks and other variables : Intelligence, Reasoning ability, Space relations, Adjustment, Personality traits and Academic achievement on 986 adolescent pupils studying in grade VI to X. The factor analysis of data by the Principal component method and then rotated by varimax showed the fourteen factors.

Padmini (1982) investigated the growth of exclusion of variables during adolescence on 200 students studying in grades VI to X using 39 variables. Stating and Testing of hypotheses, Intelligence, HSPQ and Aptitude tests, Permutations, Combinations, Problem Sensitivity and Grasping the essence of the problem were administered and the following twelve factors were obtained using principal component method (Varimax rotation) : Language Factor, Exclusion of Variables (Testing Hypotheses), Exclusion of Variables (Stating

Hypotheses), Super Ego Strength, Group Factor of Personality, Permutations and Combinations, Mechanical Reasoning, Self Sufficiency, Age, Parmia, Ego Strength and Intelligence.

Jain (1982) conducted a study of the problem solving behaviour in Physics using 34 variables on a sample of 180 adolescent pupils studying in grade XI (Science). Using Scree test last six factors were eliminated out of the existence of the twelve factors. The following six factors were interpreted psychologically : General Schematic learning, Creative Thinking, Academic Achievement or Experience in Science, Probabilistic Reasoning, Proportional Reasoning and Piagetian Cognitive Development.

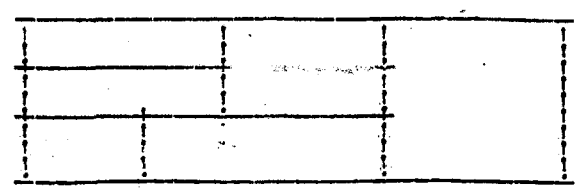
The Present Study

It had two objectives in view. First, it was to determine the mathematical structure of the problems used. The second one was to determine whether processes of thought could be derived from the individual problems and whether they appeared when data were subjected to factor analysis. The data of all the 38 variables included in the study for the entire sample (N=400) were subjected to factor analysis to study the mathematical structure of the adolescent thought. The major objective of this study was to make a comprehensive study of the mathematical structure for adolescent thoughts underlying the problems and their processes in Piagetian context. The table given below presents the total picture when seen from the above mentioned two points of view.

Table XI

Table Showing the number of Variables and the Factor Loadings of each Variable on Nine Extracted Factors

V. No.	S. No.	Problem	Process No.	Factor I	Factor II	Factor III	Factor IV	Factor V	Factor VI	Factor VII	Factor VIII	Factor IX
1.		Grade(V,VI,VII,VIII,IX)										
2.		Sex (Boys & Girls)										
3.	1.	No.1 Height Problem	1. What is the height of Mohan?			0.81						
4.	2.		2. What is the height of Sohan?			0.75						
5.	3.		3. What is the height of Shyam?			0.78						
6.	4.		4. How much is Mohan shorter than Shyam?			0.66						
7.	5.		Total Score of Problem No. 1			0.95						
8.	6.	No.2 Positive Constant Difference Problem	5. 25 ?	0.63								-0.34
9.	7.		6. ? 35	0.50								0.58
10.	8.		7. ? 40	0.84								
11.	9.		8. 40 ?	0.79								
12.	10.		9. ? 50	0.81								
13.	11.		10. 50 ?	0.69								
14.	12.		Total Score of Problem No.2	0.98								
15.	13.	No.3 Time Problem	11. How much time will you take if you go on bicycle five times at more speed than the speed on foot?				0.89					
16.	14.		12. How much time will you take if you go on bicycle from one town to the other and return to the same town?				0.89					
17.	15.		Total Score of Problem No.3				0.93					
18.	16.	No.4 Rectangle Problem	13. What is the total distance when the man goes once round the rectangle?		0.85							
19.	17.		14. What is the total distance when the man goes four times round the rectangle?		0.89							
20.	18.		15. How much time does he take for going once round the rectangle?		0.80							
21.	19.		16. Suppose he rests for a second at the end of each round. How much time does he take for completing just four rounds									0.68
22.	20.		Total Score of Problem No.4		0.93							

28.	26.		22.	How much did they spend in all?
29.	27.		23.	How much additional money was demanded by the hotel manager?
30.	28.			Total Score of Problem No.5
31.	29.	No.6 Questions inviting wrong Answer Problem	24.	A blindman with one eye can see upto a distance of 100ft How far can he see with two eyes?
32.	30.		25.	It takes 10 minutes for a boy to reach his school. How much time will it take for 10 boys if they start for the school at the same time?
33.	31.		26.	A stick is 10 cm.long.It is cut by a cm.per minute. In how much time will it be cut into 1 cm.pieces?
34.	32.		27.	Suppose a donkey has two horns. How many horns in all have eight donkeys?
35.	33.		28.	A cow is standing beside a tree.A Cord of 50 cm.is tied around her neck. Tell how far can she go for eating the grass?
36.	34.			Total Scores of Problem No.6
37.	35.	No.7 Counting Maximally Rectangle Problem	29.	(Score of problem No.7) 
38.	36.			Total Scores of all Problems

0.84
0.78 0.52

0.84

0.46

0.77

-0.64

0.36 0.45
0.74

0.43

0.42 0.45

Interpretation of Table XI

A valid need picture appears when it comes to the interpretation of various factors. Before describing this picture, it is necessary to make the following three rmearks:

1. All the individual factor loadings have been reduced to two decimal places.

2. To simplify the interpretation of factors and to draw attention only to those factor loadings which contribute significantly for determining the nature of the factor, smaller factor loadings were ignored. However, there is no definite criterion for judging how "small" a factor loading should be, to be called "small" to be ignored. Benjamin Fruchter (1967) has suggested that values of factor loadings less than .20 are, generally speaking, insignificant and hence can be ignored, whereas various other research workers have ignored values upto .300 (Vaidya, 1975). In the present study also the loadings of the different variables on the various factors having values numerically less than .300 have been ignored while interpreting the factors.

3. Usually speaking, any factor whose latent roots or eigen value is greater than one is supposed to be significant. In this study we have only used seven problems and twenty-nine processes. Naturally we should not expect more than seven factors. Since there is overlapping among problems and also among processes, it is necessary to reduce the number

of significant factors for better psychological interpretation of the same. Nine significant factors were obtained which were having eigen values greater than one. So technique suggested by Professor Cattell called the Scree-test was used. In this technique the latent roots are plotted against factor on Y axis and X axis respectively. Kink in the curve is noticed. Factors beyond the Kink on the X axis are eliminated. This was done in this study as well. The curve below showing latent roots of the significant factors shows that three factors from bottom to upwards can be easily eliminated resulting in the first six factors of distinct values.

The first nine factors, with their eigen values, percentages of 14 total variances and cumulative percentage of eigen values are shown in table XII.

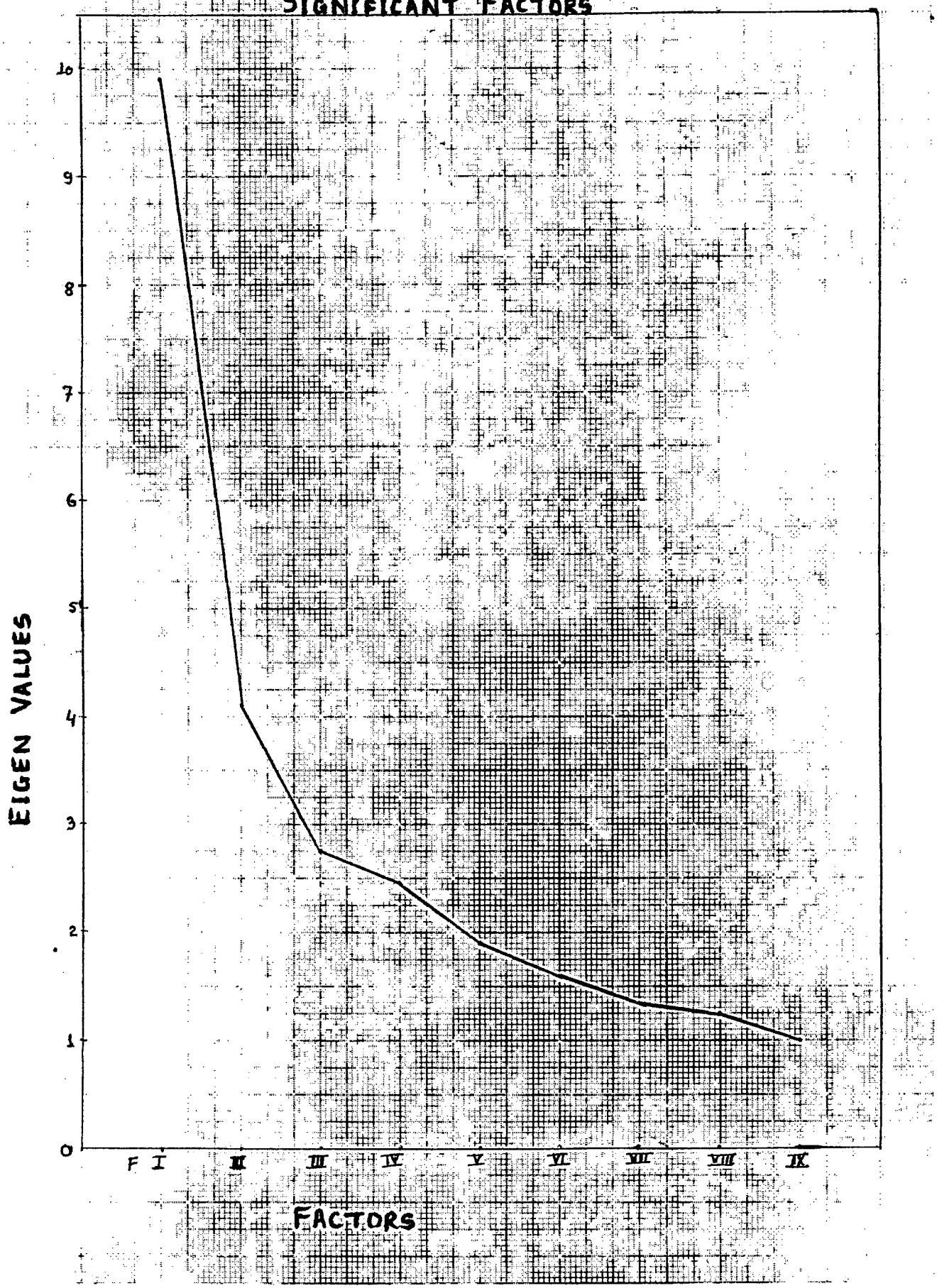
Table XII

Eigen Values, Percent of Variance and Cumulative Percentage for the Nine Factors

S.No. of factor	Eigen values	Percent of variance	Cumulative percentage of Eigen value
I	9.90	27.5	27.5
II	4.09	11.4	38.9
III	2.75	7.6	46.5
IV	2.44	6.8	53.3
V	1.91	5.3	58.6
VI	1.61	4.5	63.1
VII	1.37	3.8	66.9
VIII	1.25	3.5	70.4
IX	1.03	2.9	73.2

GRAPH No. 7

GRAPH SHOWING THE EIGEN VALUES OF THE SIGNIFICANT FACTORS



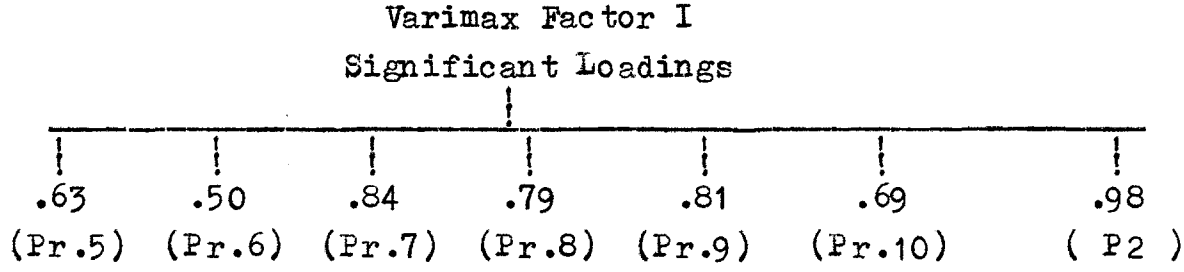


Figure 1

First Factor

The variable : total score on problem 2 has highest loading on factor I. There are no other variables which have significant loadings on factor I. So this factor is interpreted as using constant difference (Positive).

It has high educational value when '0' errors are to be applied in various scientific measurements.

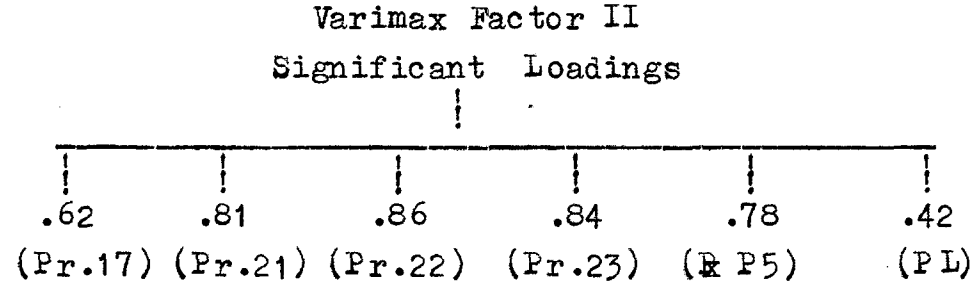


Figure 2

Second Factor

Here the highest loading of the process No.22 of the hotel problem is on this factor. The process No.23 has the next highest loading on this factor. This factor can be interpreted as summation factor in school problems.

Varimax Factor III
Significant Loadings

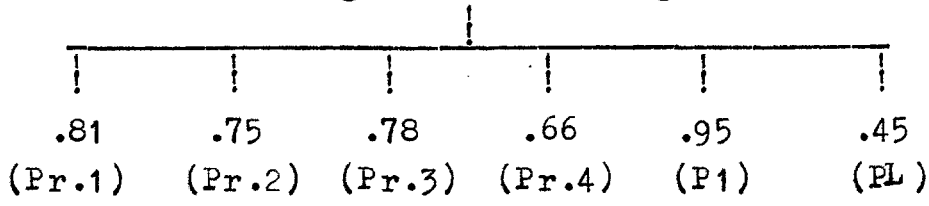


Figure 3

Third Factor

Here the highest loading is that of the total score of problem No.1. This problem deals with applying comparative difference when one of the basic heights figure is given. This factor can be interpreted as applying flexible or comparative differences two way complex problems.

Varimax Factor IV
Significant Loadings

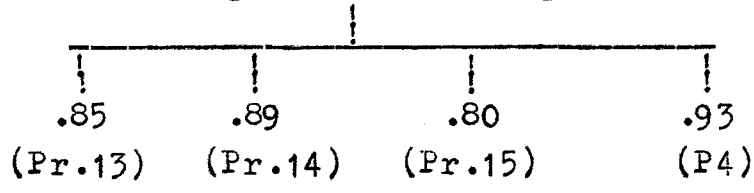


Figure 4

Factor Four

The rectangle problem needs lot of insight to solve it. So this factor is interpreted as insight factor.

Varimax Factor V
Significant Loadings

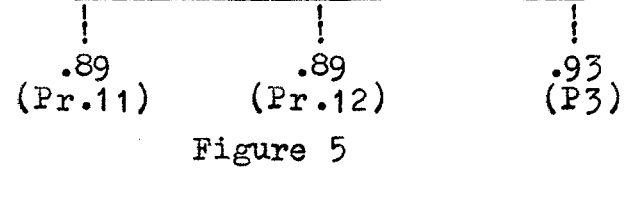


Figure 5

Fifth Factor

Here the problem involves proportion. So this factor is interpreted as proportional reasoning.

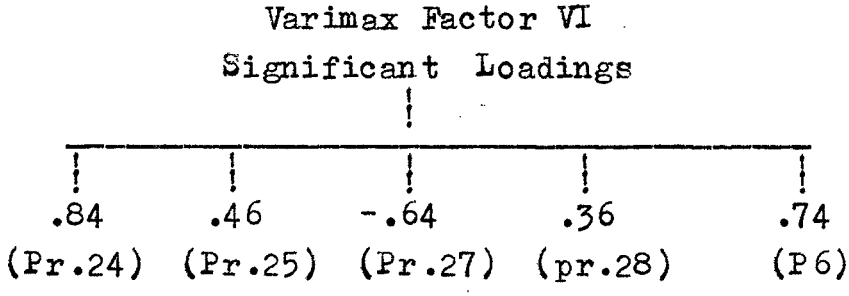


Figure 6

Sixth Factor

In this problem, the ways processes of thought really invited wrong answers as the scoring is reversible, this factor is interpreted as grasping the essence of the problem.

But what do the above interpretation show? First each problem has its own factorial structure. Second, processes of thought can be derived from problems through factor analysis. Third, the necessity of using the Scree-test is essential. Fourthly, formal thought within the context of this study comprises : Using constant difference, Using summation, Applying differences to an unknown situation insight, and, Proportional scoring-grasping essence of the problem.

Conclusion

When results of the other workers are considered. It appears their factors do not appear in this study because tests involving such factors were not included in this study. However, the factors which appears in this study also appear in the studies of Vaidya, Sandhu and Jain who have studied the structure of formal thoughts over a wide spectrum. It may however be added that the study of Jain is restricted to the study of advanced problem solving behaviour in Physics alone. So in conclusion it can be safely said that formal thought within the Piagetian context definitely requires the following behaviours :

1. Reacting to those situations which are not physically present.

2. Use of constant difference, summation, applying flexible or comparative difference, insight, proportional reasoning and grasping the essence of the problem, are some of the distinguishing marks or characteristics of formal thoughts.

Hence, hypothesis No.5 that "Using Constant difference, proportion, insight, summation, applying comparative difference and grasping the essence of the problem are not distinguishing characteristics of Piagetian thought during adolescence" is rejected.